O(n) compression of bounded M-Sum constraints in "Traffic Engineering with Forward Fault Correction"

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Microsoft Research Technical Report

1. INTRODUCTION

In our recently published paper [1], we propose a method which leverages bubble sorting network to transfer " Bounded M-sum constraint" into an efficient format. This method can use $O(kn)$ constraints to efficiently express the original $O(\binom{n}{k})$ constraints without losing any optimity in the objective of TE.

In this technical report, we will present a new method which can compress the original $O(\binom{n}{k})$ constraints into $O(n)$ constraints without losing the optimity of the TE objective.

2. BOUNDED M-SUM CONSTRAINT

In section 4.4, we show that FFC constraints on both control and data plane can be transferred into a single constraint format, which is called "Bounded M-sum constraint ".

DEFINITION 1 (BOUNDED M-SUM CONSTRAINT). *Given a set of variables $X = \{x_1, \ldots, x_n\}$, and let $x^{(i)}$ be the ith largest element in X, this constraint requires that:*

$$\sum_{i=1}^{M} x^{(i)} \leq B \tag{1}$$

where B is a bound.

In other words, if we can make sure that (1) is true in the final TE solution, the TE solution then is surely to satisfy the FFC requirements.

In this technical report, we show how to compress the number of constraints in (1) from $O(\binom{n}{M})$ to $O(n)$ with a primal-dual technique in robust linear programming context.

3. THE SOLUTION

Define function $f(x_1, x_2, \ldots, x_n) = f(X) = \sum_{i=1}^{M} x^{(i)}$ as the sum of the largest M elements in X.

Given a fixed $X = \{x_1, \ldots, x_n\}$, $f(X)$ is the optimal objective value of the following LP:

$$\text{maximize} \sum_{i=1}^{n} x_i * y_i \tag{2}$$

$$\text{s.t.} \sum_{i}^{n} y_i = k \tag{3}$$

$$\forall i \in [1, n] : y_i \in [0, 1] \tag{4}$$

where $x_1, \ldots x_n$ are parameters and $y_1, \ldots, y_n$ are variables.

The dual of this problem is:

$$\text{minimize } kz + \sum_{i=1}^{n} g_i \tag{5}$$

$$\text{s.t. } \forall i \in [1, n] : z + g_i \geq x_i \tag{6}$$

$$\forall i \in [1, n] : g_i \geq 0 \tag{7}$$

Because $f(X)$ is the objective value of the primal optimal, it is also the objective value of the dual optimal (strong duality). Therefore, the constraint $f(X) \leq B$ is equivalent to the following set of linear constraints:

$$kz + \sum_{i=1}^{n} g_i \leq B \tag{8}$$

$$\text{s.t. } \forall i \in [1, n] : z + g_i \geq x_i \tag{9}$$

$$\forall i \in [1, n] : g_i \geq 0 \tag{10}$$

, because $kz + \sum_{i=1}^{n} g_i \leq B$ is sufficient to guarantee that $\min\{kz + \sum_{i=1}^{n} g_i | \forall z, g_i\} \leq B$.

There are only about $2n$ constraints, instead of O(Mn) if we use sorting network as in [1].

4. REFERENCES

[1] H. H. Liu, S. Kandula, R. Mahajan, M. Zhang, and D. Gelernter, "Traffic Engineering with Forward Fault Correction," in *SIGCOMM'14*.